New Equivalent Circuits for Inhomogeneous Coupled Lines with Synthesis Applications

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Abstract —**Previously published equivalent circuit representations of parallel-coupled lines in an inhomogeneous medium by Zysman and Johnson are very complicated, and quite unsuitable for application in distributed filter synthesis. This defect was remedied in a previous (1984) conference paper by resynthesizing the circuits in a new and physically meaningful form. The theory is now extended to give an approximate but highly accurate synthesis of a 3:1 bandwidth inhomogeneous distributed high-pass filter realized in suspended substrate stripline. The new procedure is almost purely analytic, and computer-aided design is required only for fine tuning adjustments. Theoretical feasibility of designing such filters for upper pass bandwidths of greater than 8:1 is demonstrated.**

I. The New Equivalent Circuits

EQUIVALENT circuits for inhomogeneous coupled-line circuits have been presented by Zysman and Johnson [1]. Here the term *inhomogeneous* means simply that the even and odd modes have different phase velocities and different electrical lengths, θ_e and θ_o respectively. These circuits are exact and suitable for circuit analysis, but several of the representations are inconvenient for cascade synthesis procedures. Four of the most popular circuits are shown in Fig. 1, column 1, with the Zysman–Johnson equivalent circuits given in column 2. The new equivalent circuits are given in column 3, and were presented previously [2]. They are repeated here for the purposes of clarity and wider dissemination, and are evidently more suitable for identification as circuit elements in a synthesis procedure.

Thus in circuit (a) the resonant part of the circuit is extracted and clearly identified as series open-circuited stub of impedance Z_{0o} and of length θ_o. The remaining part of the circuit is a series connection of two unit elements, one of impedance $Z_{0e}/2$ and length θ_e, the other of impedance $-Z_{0o}/2$ and length θ_o. Initial reaction may be that this part of the circuit is as complicated as that of Zysman and Johnson, but in some interesting practical situations $Z_{0e} \gg Z_{0o}$, and the odd mode unit element may be disregarded, at least initially. Hence the new equivalent circuit effectively separates the odd and even parts of the network and additionally results in a desirable cascaded topology. The same considerations apply to the other equivalent circuits shown in Fig. 1.

Later, it will be shown how the exact representations may be used, and a very good approximate synthesis carried out. The derivation of the equivalent circuit of Fig. 1(a) is given in the Appendix.

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The fourth column in Fig. 1 shows the degenerate homogeneous equivalent circuits, as presented by Ozaki and Ishii [3], and indicates how closely these are related to the inhomogeneous circuits of column 3.

II. Synthesis of Inhomogeneous Parallel-Coupled Line Filters

The latter equivalent circuit of Fig. 1(a) may be applied to the synthesis of simple parallel-coupled line filters, such as in Fig. 2(a), and may lead to a very satisfactory synthesis of inhomogeneous filters of this type, commonly built in microstrip. However the technique presented here is more appropriate for suspended substrate where $Z_{0e} \gg Z_{0o}$, and the example to be presented is the synthesis of the pseudoelliptic suspended substrate stripline high-pass filter shown in Fig. 2(b). Only rudimentary design procedures have been published for this filter [4], and most workers have resorted to computer-aided design (optimization) techniques.

The new design procedure is based on the homogeneous distributed prototype filter, shown in Fig. 3. The transfer function may be written down using standard transformed-variable Chebyshev theory [5]. The appropriate transformed variable Z is defined in terms of the Richards variable S by the conformal transformation

$$Z^2 = 1 + S^2/\Omega_c^2 \tag{1}$$

where at real frequencies

$$S = j\tan\theta \tag{2}$$

θ being the commensurate electrical length, and the equiripple passband edge θ_c of the high-pass filter corresponds to

$$\Omega_c = j\tan\theta_c. \tag{3}$$

The prototype shown in Fig. 3 is of order n (odd), where there are $(n-1)/2$ shunt Fosters (i.e., finite pole-producing sections) and $(n+1)/2$ contributing unit elements. The series capacitors (actually series open-circuited stubs) contribute a single ordered pole of attenuation at zero frequency. The shunt Fosters each contribute a double-ordered pole at $S = j\Omega_p$, the unit elements each a single-ordered pole at $S = j1$, and the series capacitors a single-ordered pole at $S = 0$. It can be shown [5] that the Chebyshev approximation problem is solved by forming the function

$$f(Z) = \prod_{i=1}^{n} \sqrt{\frac{Z_i + Z}{Z_i - Z}} \tag{4}$$

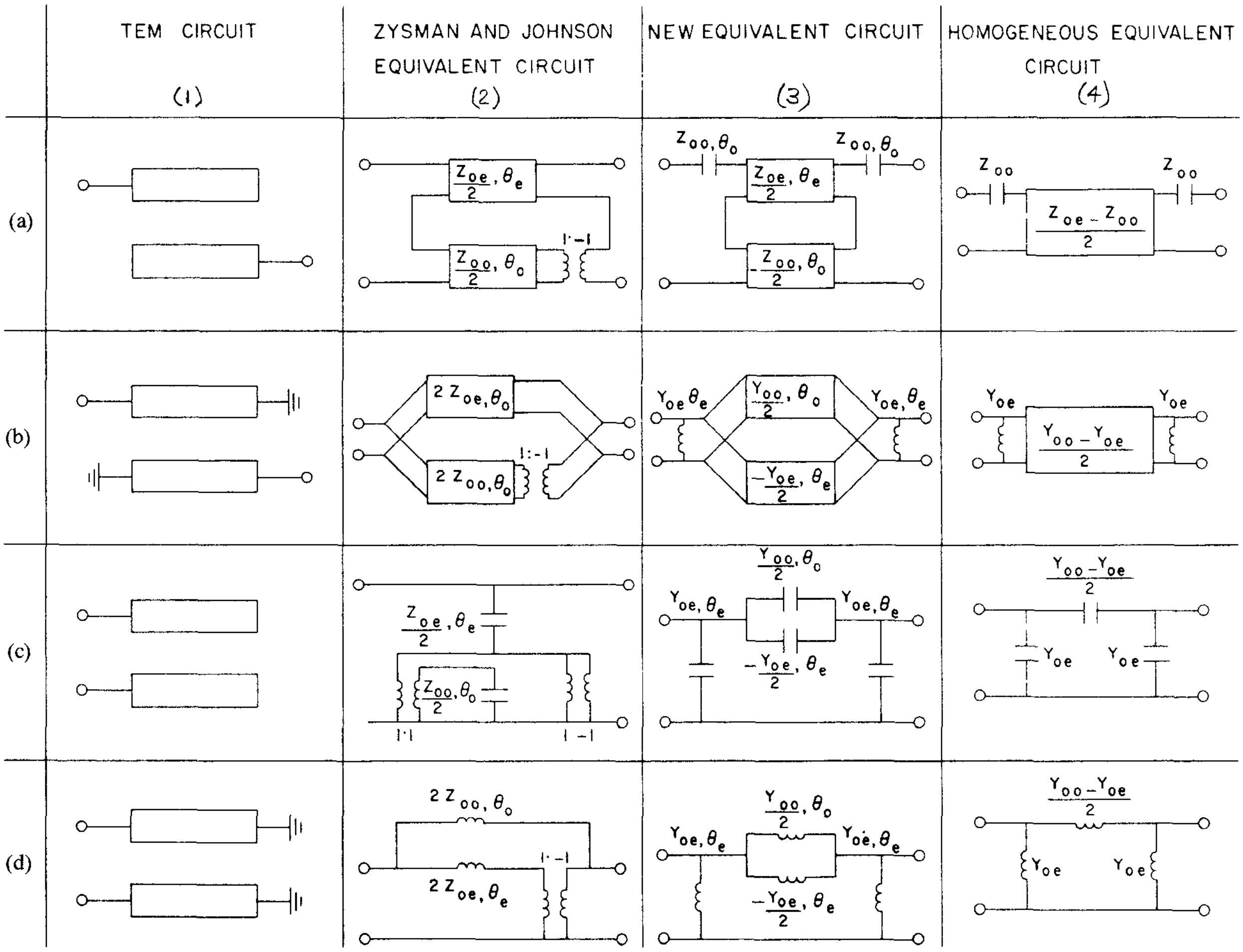

Fig. 1. Old and new equivalent circuits for inhomogeneous parallel-coupled lines (capacitors represent open-circuited stubs, inductors short-circuited commensurate stubs).

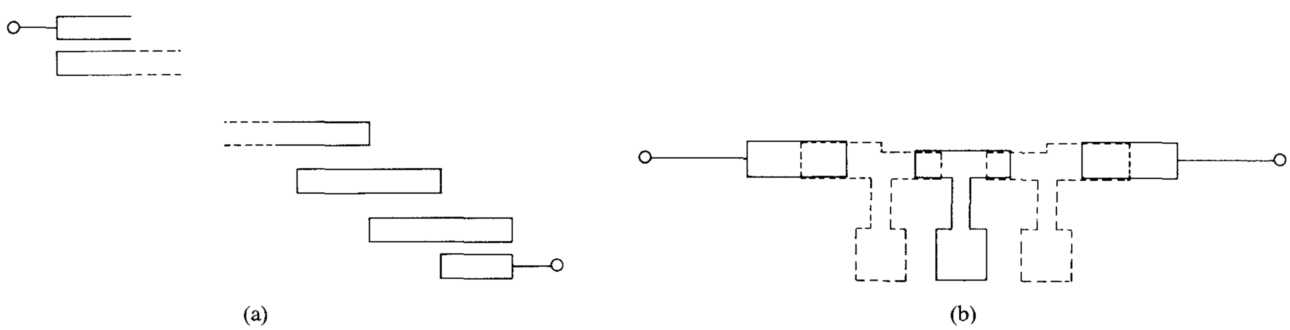

Fig. 2. (a) Parallel-coupled line filter. (b) Pseudoelliptic high-pass filter. The solid and dashed lines represent conductors on either side of a suspended substrate.

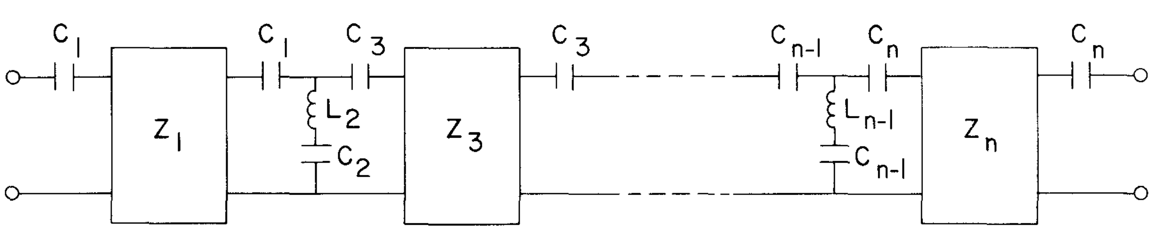

Fig. 3. Homogeneous distributed high-pass filter.

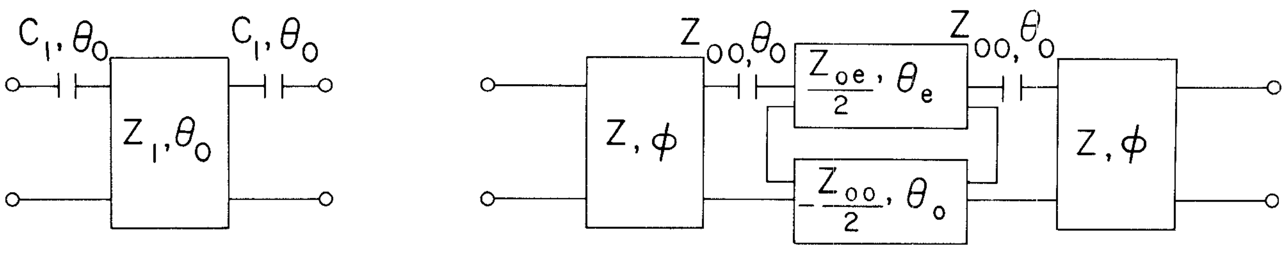

Fig. 4. (a) Homogeneous prototype section and (b) modified inhomogeneous coupled-line section.

where

$$\prod_{i=1}^{n}(Z_i+Z)=(1+Z)$$

$$\cdot\left(\sqrt{1-\frac{\Omega_p^2}{\Omega_c^2}}+Z\right)^{(n-1)/2}\left(\sqrt{1+\frac{1}{\Omega_c^2}}+Z\right)^{(n+1)/2}. \quad (5)$$

The equiripple insertion loss function is then

$$\frac{P_0}{P_L}=1+\epsilon^2\left|\frac{f(Z)+f(-Z)}{2}\right|^2. \quad (6)$$

The synthesis cycle commences with the complete extraction of a unit element, followed by partial extraction of a series capacitor of value such that a shunt Foster (i.e., pole) can be extracted next. Part of the series capacitor is then transferred back across the unit element using a Kuroda transformation, such that the two capacitors are identical on either side of the unit element, as indicated in Fig. 3. The shunt Foster is extracted, and the cycle repeated to form the complete prototype network. Synthesis of this filter using commercially available computer programs has been described previously [6]. However a specialized dedicated program is much faster and far more convenient.

The homogeneous prototype is now converted into the practical inhomogeneous filter using a few simple concepts. The commensurate line length is θ_p, and the shunt LC's are directly realizable as compound shunt stubs. The unit elements and associated series capacitors are realized using the equivalence shown in Fig. 4, described in more detail as follows.

In theory the odd mode impedance Z_{0o} may be chosen to equal $1/C_1$, giving exact equivalence for the series elements. C_1 is normally of quite large value, giving the desired low value for Z_{0o}. The impedances of the unit elements Z_1 are always approximately unity, typically lying in the range 0.9 to 1.1. Since in suspended stripline there is very tight coupling between lines, $Z_{0e} \gg Z_{0o}$, and the even mode unit element dominates in the series-connected pair of Fig. 4(b). Hence it is possible to select the value of Z_{0e} to satisfy the equation

$$\frac{Z_{0e}-Z_{0o}}{2}=Z_1. \quad (7)$$

Typical normalized values here are $Z_{0e}=2.3$, $Z_{0o}=0.3$,

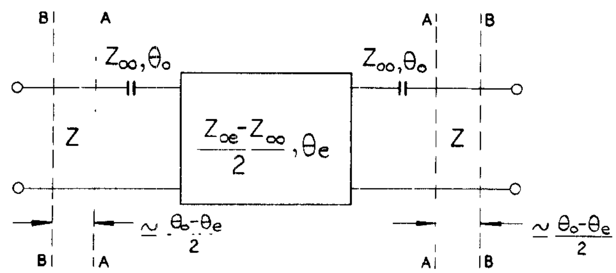

Fig. 5. Approximate equivalent circuit of the inhomogeneous coupled line section.

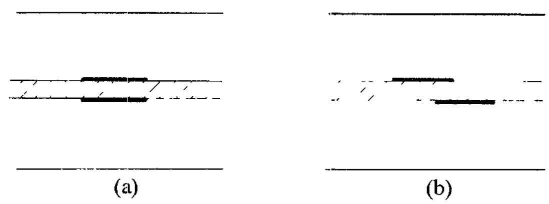

Fig. 6. Coupled lines in suspended substrate stripline. (a) Completely overlapped. (b) Partially overlapped.

$Z_1=1$. The pair of series unit elements of Fig. 4(b) may be approximated therefore by a simple unit element of impedance given by (7) but having an electrical length of θ_e, because the even mode impedance dominates in this region of the equivalent circuit, which is shown in Fig. 5. We have chosen the value $(Z_{0e}-Z_{0o})/2$ in (7) rather than $Z_{0e}/2$ to ensure that this approximate circuit degenerates to the exact homogeneous circuit of Fig. 1(a), column (4), as the even and odd mode phase velocities approach equality. In practice there is considerable flexibility of choice, as shown in Section IV.

So far the values of Z_{0e} and Z_{0o} have been determined in addition to the basic commensurate electrical length θ_o. However the effective electrical length of the unit element shown in Fig. 5 must be set at θ_e, which is too short since $\theta_e<\theta_o$ in a pair of tightly coupled suspended striplines. A cross section through such a coupled line pair is shown in Fig. 6(a), which depicts complete overlap, a configuration which may be analyzed precisely. The partially overlapped case of Fig. 6(b) allows independent control over Z_{0o} and Z_{0e}, and design information is becoming available, e.g. [7]. For inhomogeneous coupled lines of this type it is obvious that $\theta_e<\theta_o$ since most of the even mode field is in the air region and little in the dielectric, whereas the opposite is true for the odd mode.

of ϕ from (20). In this case it has been established that the line width has been chosen to be too wide.

Example

It was desired to build a high-pass filter to pass 6–18 GHz with insertion loss <1 dB and VSWR <1.6:1 (12.7 dB return loss). The rejection was to be >60 dB below 5.3 GHz. The prototype used contained four coincident finite frequency poles, one pole at dc, and five unit elements, i.e., the same topology as shown in Fig. 7. Comparison with results given in [8] demonstrates that the contributing unit elements give a very significant improvement in stopband performance, e.g. equivalent to at least one extra finite frequency pole. The parameters of the prototype are ripple VSWR of 1.2:1, pole angle of 30°, band edge angle of 35.294°, and stopband ripple level of 63.67 dB, which first occurs at $\theta = 31.92°$ near the upper passband. Hence if the passband edge is set at 5.9 GHz the stopband edge will be at $(31.92/35.294) \times 5.90 = 5.38$ GHz.

The inhomogeneous design derived from this prototype showed good theoretical performance, having return loss better than 20 dB up to 18.5 GHz. It was constructed in suspended substrate stripline with a ground plane spacing of 0.10 in., substrate thickness of 0.015 in., and dielectric constant of 2.22. The dimensions in the filter are significant compared to a quarter-wavelength at 18 GHz, and T junction, end effects, and dispersion must be taken into account accurately, but the structure has a high unloaded Q. The length of the filter is 1.05 in. (excluding the housing). The performance of a typical unit (Fig. 10) indicates that at these high frequencies the return loss is worse than that of the ideal inhomogeneous filter due to a variety of factors, but is within specification and better than 17 dB over most of the band, with one dip at about 13 dB. The insertion loss is better than 0.75 dB over 6–18 GHz. Measurements above 18 GHz confirmed that the VSWR behaves in accordance with the theory, as indicated in Fig. 10. The stopband rejection also is in almost exact agreement with theory. These results were obtained with slight adjustments to the stub lengths only.

Physical dimensions of the coupled-line regions were obtained from several well-known publications, and a good summary may be found in a paper by Bhartia and Pramanick to be published later this year [9].

V. Conclusions

New equivalent circuits for inhomogeneous coupled-line sections enable cascade synthesis procedure to be applied to the design of very broad band inhomogeneous filters. The method has been implemented in the design of inhomogeneous high-pass filters constructed in suspended substrate stripline, where the condition $Z_{0e} \gg Z_{0o}$ applies. It would be interesting to apply the equivalent circuit concept to the design of microstrip filters where the above condition does not apply. Here it should be possible to predict the results obtained by Easter and Merza [10].

It would be interesting also to extend the concept to asymmetric inhomogeneous coupled lines, where the equivalent circuits derived by Tripathi [11] could conceivably be resynthesised in a similar way to give more meaningful physical representations.

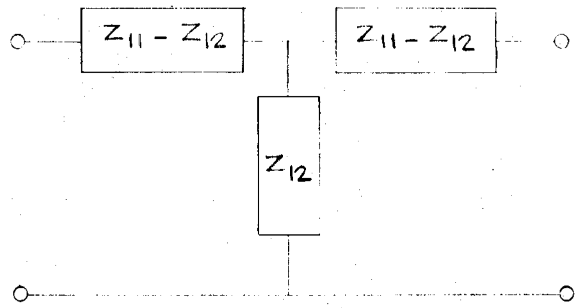

Fig. 11. Equivalent T network of a two-port impedance matrix.

Appendix

The transfer matrix of the inhomogeneous coupled line section of Fig. 1(a) is given by Zysman and Johnson [1] as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{A1}$$

where

$$A = D = \frac{Z_{0e}\cot\theta_e + Z_{0o}\cot\theta_o}{Z_{0e}\operatorname{cosec}\theta_e - Z_{0o}\operatorname{cosec}\theta_o} \tag{A2}$$

$$B = \frac{j}{2}\,\frac{Z_{0e}^2 + Z_{0o}^2 - 2Z_{0e}Z_{0o}(\cot\theta_e\cot\theta_o + \operatorname{cosec}\theta_e\operatorname{cosec}\theta_o)}{Z_{0e}\operatorname{cosec}\theta_e - Z_{0o}\operatorname{cosec}\theta_o} \tag{A3}$$

$$C = j\frac{2}{Z_{0e}\operatorname{cosec}\theta_e - Z_{0o}\operatorname{cosec}\theta_o}. \tag{A4}$$

The impedance matrix is

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{12} & z_{11} \end{bmatrix} \tag{A5}$$

where

$$z_{11} = \frac{A}{C} = -j\frac{Z_{0e}\cot\theta_e + Z_{0o}\cot\theta_o}{2} \tag{A6}$$

$$z_{12} = \frac{1}{C} = -j\frac{Z_{0e}\operatorname{cosec}\theta_e - Z_{0o}\operatorname{cosec}\theta_o}{2}. \tag{A7}$$

This has a T equivalent circuit consisting of $z_{11} - z_{12}$ as the series arms and z_{12} as the shunt arm, as shown in Fig. 11. Inspection of (A6) shows that it is advantageous to extract an open-circuited stub of impedance Z_{0o} and length θ_o from each side, since the remaining series arm becomes $z_{11}' - z_{12}$, where

$$z_{11}' = z_{11} - (-jZ_{0o}\cot\theta_o) = -j\frac{Z_{0e}\cot\theta_e - Z_{0o}\cot\theta_o}{2}. \tag{A8}$$

Noting that the impedance matrix of a unit element of impedance Z is

$$-jZ\begin{bmatrix} \cot\theta & \operatorname{cosec}\theta \\ \operatorname{cosec}\theta & \cot\theta \end{bmatrix} \tag{A9}$$

then we see that the remaining impedance matrix defined by z'_{11} and z_{12} represents the series connection of a unit element of impedance $\frac{1}{2}Z_{0e}$ and electrical length θ_e with one of impedance $-\frac{1}{2}Z_{0o}$ and electrical length θ_o. This completes the derivation of the equivalent circuit of Fig. 1(a), column 3.

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